$$\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x} \longrightarrow \frac{3}{\sin 3x} \cdot \frac{\sin 4x}{1} \cdot \frac{x}{x} \cdot \frac{4}{4}$$

$$\lim_{x\to 0} \frac{\sin x}{x}$$

$$\longrightarrow \frac{3x}{\sin 3x} \cdot \frac{\sin 4x}{4x} \cdot 4$$

$$\longrightarrow \frac{\sin 4x}{4x} \div \frac{\sin 3x}{3x} \cdot 4$$

$$\lim_{x\to 0} \frac{\sin 4x}{4x} \div \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0} 4$$

$$\downarrow$$

$$1 \div 1 \cdot 4 = \boxed{4}$$

(5) $V = \frac{1}{3}\pi r^2 h$

$h = d = 2r$

$V = \frac{1}{3}\pi r^2 (2r)$

$V = \frac{2}{3}\pi r^3$

$\frac{dV}{dr} = 3 \cdot \frac{2}{3}\pi r^2$

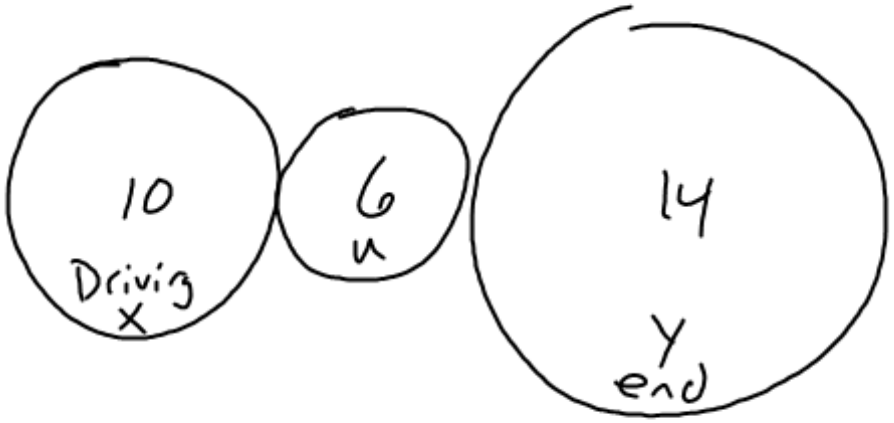
$\frac{dV}{dr} = 2\pi r^2$

$\left.\frac{dV}{dr}\right|_{r=4} = 2\pi(4)^2 = 32\pi \text{ in}^3/\text{in}$

Use $\underline{3}$ at a time

- turn one, your "driving gear", and keep track of how much the middle gear and end gear turn.

- Come up with a rule for finding how much the end gear turns per turn of the driving gear.



$$\frac{dy}{dx} = \frac{14}{10}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \frac{14}{10} = \frac{14}{6} \cdot \frac{6}{10}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$y = \sin(3x^2+5)$ $\quad u = 3x^2+5$

$y = \sin(u)$ $\quad \frac{du}{dx} = 6x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$\frac{dy}{du}$: $\cos u$ $\quad \frac{du}{dx}$: $6x$

$\cos(3x^2+5) \cdot 6x$

$6x\cos(3x^2+5)$

$$y = \sqrt{(3x^2+5)}$$

$$y = \sqrt{u} \rightarrow u^{\frac{1}{2}} \qquad u = 3x^2+5$$

$$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}} \qquad \frac{du}{dx} = 6x$$

$$\frac{du}{dx} = 6x$$

$$\frac{dy}{dx} = 6x \cdot \frac{1}{2\sqrt{3x^2+5}} = \frac{3x}{\sqrt{3x^2+5}}$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Try

① $\cos(t^2+1)$ → $-2t\sin(t^2+1)$

② $\sin^5(x)$ → $(\sin(x))^5$ → $(u)^5$

$\frac{dy}{du} = 5u^4$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{dx} = \cos x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$5(\sin(x))^4 \cdot \cos x$

Homework

Sect. 3.6

#1-39 (about $\frac{1}{2}$)